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INHOMOGENEITIES IN THE EARLY UNIVERSE*

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ABSTRACT

In big-bang cosmology it is traditional to assume that before decoupling, matter was so tightly bound to radiation as to be prevented from forming any permanent structure. In particular, galaxies were difficult to form before t_D.

It is the goal of this paper to point out that the previous arguments are based on linearized theories for both radiation and matter, as well as for their interaction, and that if nonlinear effects are taken into account, the situation can change considerably.

For matter, we shall present several nonlinear Lagrangians and discuss in particular the salient features of the most typical equation describing nonlinear phenomena, the Korteweg–deVries (KdV) equation, and the existence among its solutions of *solitons*, self-bound packets of energy. We shall then present our first model, based on the possibility of creating a cascade of solitons from an initial perturbation. We shall show how an initial soliton can, through the nonlinear action of the terms contained in the KdV equation, fragment into many others, the number of them being proportional to the amplitude of the original one.

We shall then discuss our second model, based on the interaction of matter with an intense radiation field. For the electromagnetic field, we shall discuss the Euler-Heisenberg nonlinear Lagrangian, and the changes in the refractive index of the medium.

Lacking a general description for the nonlinear interaction of matter and radiation, we shall limit ourselves to the case of matter interacting with an almost monochromatic photon gas. It will be shown that an effective mechanism exists whereby radiation and matter do not mix, if the radiation field has a nonuniform intensity, i.e., $\nabla I(r) \neq 0$. The matter so separated from radiation behaves independently of the surrounding radiation.

It is also shown that the conditions are appropriate for the interface between matter and radiation to become unstable and for a turbulent layer to form. After the previous arguments are translated into the proper cosmological framework, we propose to interpret the portions of matter so isolated as being closely related to the delayed cores forming the core of protogalaxies.

Subject headings: cosmology — galaxies: formation

I. INTRODUCTION

One of the most intriguing problems in modern cosmology is centered on galaxies: formation, evolution, morphological differences, relation to strong radio sources, QSOs, etc., are but a few of the topics whose interconnections are still poorly understood.

One of the favorite and most promising lines of research is due to von Weizsäcker, who suggested that galaxies be looked at as remnants of an originally turbulent medium (Dallaporta 1972; Nariai and Tomita 1971; Rees 1970).

Turbulence is unfortunately not a fully understood phenomenon even when the medium is incompressible.

Perhaps even more serious is the problem of the origin of cosmic turbulence. It is somewhat surprising that none of the methods of generating laboratory turbulence have ever been proposed as a possible driving mechanism. Ozernoi and collaborators (Ozernoi and Chernin 1968, 1969; Ozernoi and Chibishov 1970) suggested that before decoupling the photon gas was in a state of high vorticity and drove matter into a turbulent state when matter and radiation decoupled. If this is the case, then one must explain how the photon gas itself got to be turbulent; and as far as the author knows, no generally acceptable answer has yet been given. Most of the studies on turbulence have dealt with incompressible turbulence; and even though a fully satisfactory theory is still lacking, one possesses several reliable empirical facts (like the Kolmogorov spectrum, the permanence of big eddies, etc.) on which one can safely rely. Although it is by no means clear how the whole set of phenomena would change in an expanding medium, the generally accepted working hypothesis is that one can actually work out a series of reliable results.

For the case of compressible turbulence, the situation is very poorly understood even at the laboratory level (i.e., without expansion), and simple tricks like the one of changing the power of the Kolmogorov spectrum are indeed

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idle exercises. The fact that after the decoupling the velocity of sound decreased enough to render the medium compressible and therefore bound to lose energy in sound, has recently been put forward as a difficulty of the turbulent model (Jones 1973; Nariai 1973).

Peebles (1973) has analyzed these amid several other aspects of the turbulent model, and the reader is referred to his paper for a detailed balance of the pros and cons of this model.

An alternative model calling for the growth in time of an initial perturbation has been thoroughly studied since it was first proposed by Jeans. Lifshitz (1946) showed that for at least two types of equations of state $p = c_s^2\epsilon(c_s^2 = 0, \frac{1}{3})$ the expansion of the Universe slows down the (time) growth of the initial fluctuation from an exponential form to a power law, thus rendering the mechanism less promising. Adams and Canuto (1975) have recently shown that the Lifshitz equations can be solved analytically for any equation of state of the form $p = c_s^2\epsilon$ and that the growth is not faster than a power law. Peebles and Yu (1970) have nevertheless shown that the mechanism can still work if the perturbations were initially large.

Lifshitz's analysis, being a linearization of Einstein's equations, cannot fix the original perturbation $\delta(0)$ that would grow later in time. In the usually accepted analysis, one considers $\delta(0)$ as being given $\delta(0) \approx N^{-1/2}$, N being the number of particles under consideration. The previous formula is valid for a perfect gas and is invalid if there is a phase transition (Adams and Canuto 1975).

Both methods have been extensively studied over the years; and if they did not provide the definitive answer, it is certainly not because the mathematical canvas on which they rest has not been more than adequately vivisected. By inspecting the rather extensive literature on the subject, one gets the impression that the two models have perhaps exhausted their fruitfulness, and that either a combination of the two or perhaps an altogether different point of view is needed.

It is the purpose of this paper to call attention to a series of nonlinear processes that are, in our opinion, endowed with all the necessary credentials to make them viable candidates for the process of generating large inhomogeneities in the early Universe, and consequently relevant to the galaxy formation process.

None of the ideas and formulae presented in what follows has yet been investigated to the same extent as the turbulence and/or the growth of primordial fluctuations.

We shall discuss two rather different possibilities, both based on the existence of highly nonlinear phenomena. We must point out that we shall base our considerations upon a series of empirical facts well documented both theoretically and experimentally.

1. The first proposal is based on the fact that an initial hydrodynamic wave of velocity U_0 traveling through a warm plasma generates a localized *soliton* or solitary wave. We propose to use it as an initial perturbation for the Korteweg-deVries (KdV) nonlinear equations that will generate a *cascade of solitons*, each of them representing a well spatially defined density perturbation.

2. The second model is based on the existence of what we might call a "laser effect." It demands a gas of quasi-monochromatic photons with a nonuniform intensity, $\nabla I(r) \neq 0$. We shall discuss these two points more extensively later. For the time being we want to mention the physical significance of the results. The valleys created by the photon gas must be viewed as pockets where matter can be isolated. Since it will be shown that under appropriate conditions the refractive index can be imaginary, these pockets of matter are indeed separated dense primeval portions of matter, thus creating a clearly inhomogeneous Universe of the Swiss cheese type discussed 30 years ago by Einstein and Straus (1945).

In what follows we shall present the general aspects of the nonlinear phenomena we shall later employ, i.e., the change of the index of refraction of the vacuum in the presence of a strong electromagnetic field and the existence of solitons in matter and plasma.

II. NONLINEAR EFFECTS

From quantum electrodynamics we know that when we are in the presence of strong electric and magnetic fields the simple linear Lagrangian $L_0 = (E^2 - B^2)/2$ breaks down and must be replaced by a corresponding nonlinear one, known as the Euler-Heisenberg Lagrangian. The nonlinear effects become important when B and E are comparable with B^* and E^*, where

$$B^* = \frac{m^2c^3}{e\hbar} = 4.41\ 10^{13}\ \text{gauss}\,. \tag{1}$$

By using the law of flux conservation and a value of 10^{-6} for the present magnetic field strength, we obtain for the magnetic field at an early time $t \approx 1$ s

$$B(t_2) = 10^{-6}\left(\frac{R_1}{R_2}\right)^2 = 10^{-6}\left(\frac{T_2}{T_1}\right)^2 = 10^{-6}\left(\frac{10^{10}}{3}\right)^2 \approx 10^{13}\ \text{gauss}\,, \tag{2}$$

$$T(t) = 10^{10}t^{-1/2}\,.$$

At earliest times, we expect even higher values of B and therefore a full nonlinear regime. One of the most immediate consequences of a nonlinear regime is that a photon with energy greater than $2mc^2$, traveling an empty region of space on which acts a strong magnetic field, will split into an electron-positron pair. For instance, a 10^{14} eV photon will travel less than 1 cm in a 10^6 gauss field before splitting into a pair. Such a phenomenon is clearly not included in Maxwell equations. The correction L_1 to the linear Lagrangian L_0 has been calculated by Euler-Heisenberg and Weisskopf with the result

$$L_1 = \alpha/4(F_{\mu\nu}{}^2)^2$$

with

$$\alpha = e^4\hbar/360\pi^2 m^4 c^7 \,. \tag{3}$$

The Euler-Heisenberg treatment is valid as long as the field changes only slightly over distances of the order of $\hbar/mc$ and times of the order of $\hbar/mc^2$. Higher order terms are clearly needed if $B > B^*$. The full expression for L can be found in Akhiezer and Beresteski (1965).

In the presence of a strong electromagnetic field, the refractive index of vacuum is changed from one to

$$n(\omega) = 1 + \frac{\alpha}{\pi}\left(\frac{B}{B^*}\right)^2 [N(x) + i\pi T(x)/x] \,. \tag{4}$$

The function $N(x)$ is proportioned to $x^{-4/3}$ for $x \to \infty$, and is $\sim 10/45$ for $x \to 0$, where

$$x = \frac{\hbar\omega}{mc^2}\frac{B}{B^*} \,. \tag{5}$$

The imaginary part of $n(\omega)$ represents the absorption coefficient that determines the mean free path. The function $T(x)$, plotted in Figure 3.2a of Toll (1952), has the following behavior:

$$\begin{aligned} T(x) &\approx \exp(-4/3x) \quad (x \to 0) \\ &\approx x^{-1/3} \qquad\quad\;\; (x \to \infty) \,, \end{aligned} \tag{6}$$

the coefficient of proportionately being of the order of unity.

For us, the most important result is the fact that the real part of the refracting index can become negative, i.e.,

$$n(\omega) = 1 - \frac{\alpha}{\pi}\left(\frac{B}{B^*}\right)^2 N(x) < 0 \tag{7}$$

if

$$\omega < \omega_{cr} \equiv \frac{mc^2}{\hbar}\left(\frac{B}{B^*}\right) \,,$$

as can be easily deduced from the previous equations.

This is true in vacuum. When matter is also considered, the situation becomes much more complex and we know of only one example that can be treated exactly—that of an almost monochromatic photon gas. We shall treat that in § IV.

For matter itself, even though we do not have a measure of the nonlinear effects as simple as the one for the magnetic field, we cannot expect linear field equations to hold at arbitrarily high densities as the ones we encounter at early times in the history of the Universe.

We shall therefore assume that L_m, the Lagrangian for matter, is also nonlinear. There are several examples of nonlinear equations discussed in the literature, the most widely known being the one of Born-Infeld (1934):

$$\phi_{xx}(1 - \phi_t{}^2) + 2\phi_x\phi_t\phi_{xt} - (1 + \phi_x{}^2)\phi_{tt} = 0 \,, \tag{8}$$

as well as the nonlinear Schrödinger equation (Benney and Newell 1967):

$$\phi_{xx} + i\phi_t + \lambda\phi^2\phi = 0 \,, \tag{9}$$

or its field-theoretical analog proposed by Heisenberg (1966):

$$\gamma_\mu\partial_\mu\psi + l^2\gamma_\mu\gamma_5 < \bar{\psi}\gamma^\mu\gamma_5\psi > \psi = 0 \,. \tag{10}$$

In what follows we shall not discuss any of the previous equations of motion, but rather the so-called Korteweg-deVries equation (Scott, Chu, and McLaughlin 1973):

$$U_t + \alpha U U_x + \beta U_{xxx} = 0 \,, \tag{11}$$

where α and β are two coefficients. The KdV equation has undoubtedly been studied more extensively than other nonlinear equation since it has appeared in a number of seemingly unrelated problems, like the study of shallow water waves (when it was originally derived), hydromagnetic waves in plasmas, ion-acoustic waves, and finally nonlinear lattices (Toda 1975).

The KdV equation has come to represent the quintessential nature of nonlinearity; and even though the results that follow are derived from its analysis, there seems to be no doubt that they are indeed more general in character.

Equation (11) with U_{xxx} replaced by U_{xx} is known as the Burgers equation, and it admits an exact general solution. No such exact solution is known for the KdV equation, however. The only case when the KdV can be solved analytically is when the variables x and t enter not separately but in the combination

$$\xi = x - vt\,, \tag{12}$$

where v is the velocity of propagation. In this case the original KdV equation becomes

$$vU_\xi - \alpha UU_\xi - \beta U_{\xi\xi\xi} = 0\,. \tag{13}$$

It has been shown that equation (11) admits a solitary wave or *soliton*, i.e., a solution satisfying the boundary conditions

$$U(\pm\infty) = U'(\pm\infty) = U''(\pm\infty) = 0\,. \tag{14}$$

The solution is

$$U(\xi) = U_0 \operatorname{sech}^2(\lambda\xi)\,, \tag{15}$$

$$U_0 = 3v/\alpha\,, \qquad 4\lambda^2 = v\,. \tag{16}$$

Lacking a general exact solution, Zabusky and Kruskal (1965) solved the KdV equation numerically, with the following results:

1) Given an initial, $t = 0$, condition, say,

$$U(x, t = 0) = A_0 \cos \pi x\,, \tag{17}$$

and a choice of the two parameters α and β corresponding to

$$\alpha = 1\,, \qquad \beta^{1/2} = 0.022\,, \tag{18}$$

they found that the KdV equation admits at later times a train of well-separated waves, the so-called *solitons*.

2) When the solitons are allowed to interact among themselves, they emerge from the interaction with their original shape and velocities.

3) The trajectories of solitons exhibit at least one point in time where *all solitons* arrive almost in the same phase. After that, they reconstruct almost exactly the initial state.

Many other properties have been discovered during the numerical studies of the KdV equation. For our purposes, however, the aforementioned properties are sufficient. We would like to remark that properties (1) and (2) are perhaps the most important as far as our presentation is concerned. However, before ending we must still dwell on another important property of the KdV equation discussed by Gardner *et al.* (1967).

Consider the KdV equation (11) with

$$\alpha = \beta = 1\,, \qquad U(x, t) = -6\varphi(x, t)\,. \tag{19}$$

We will have

$$\varphi_t = 6\varphi\varphi_x - \varphi_{xxx} \tag{20}$$

with a given initial boundary condition specified by, say,

$$\varphi(x, 0) \equiv V(x)\,. \tag{21}$$

By the application of inverse scattering theory, Gardner *et al.* were able to show how the initial value nonlinear KdV equation can be exactly solved by linear methods. Among the various results found in this important research, we shall concentrate specifically on one, namely that the number N of admissible solitons coincides with the number of bound eigenvalues, of the differential equation

$$\psi''(x) + [\lambda_n - V(x)]\psi(x) = 0\,, \tag{22}$$

where the potential $V(x)$ is just the initial value of $\phi(x, t)$. The KdV equation solved numerically by Zabusky and Kruskal corresponds to an initial condition

$$V(x) = U(x, 0) = -\frac{A_0}{6} \cos \pi x\,, \tag{23}$$

a potential that gives rise to Mathieu functions when inserted into the Schrödinger equation (22). We must remark that the use of the terminology "Schrödinger equation" is rather inappropriate, since there are not $\hbar$'s or m's in equation (22) and the operator

$$\mathscr{L} = \partial_x^2 + U(x, 0) \tag{24}$$

comes naturally out of the analysis of the KdV equation.

We shall however keep this terminology because of its connection with the eigenvalue problem.

III. THE FIRST MODEL

Let us consider a nonrelativistic plasma made of electrons and ions of masses m_- and m_+, respectively, embedded in a uniform magnetic field B_0 along the z-axis. Suppose now there is an hydromagnetic wave traveling with velocity U_0 along the x-axis. If we formulate the problem in a system of coordinates traveling with the wave, the Boltzmann equation (for one species) will be

$$u^* \frac{\partial f^*}{\partial x^*} + \frac{e}{m}\left(E_x + \frac{v^*}{c} B^*\right)\frac{\partial f^*}{\partial u^*} + \frac{e}{m}\left(E_y - \frac{u^*}{c} B^*\right)\frac{\partial f^*}{\partial v^*} = 0\,, \tag{25}$$

where

$$f^*(x^*, u^*, v^*)$$

is the density of that species and u^* and v^* are the x and y components of the velocity. $E_y = B_0 U_0$, since the electric field in the y-direction is due to transformation of coordinates. The electric field E_x is due to charge separation, and its space variation as well as that of $B^*(x)$ is governed by the Maxwell equations

$$\frac{\partial E_x}{dx^*} = e^2 c\left(\iint f_+{}^* du^* dv^* - \iint f_-{}^* du^* dv^*\right),$$

$$\frac{dB}{dx} = e\left(\iint v^* f_+{}^* du^* dv^* - \iint v^* f_-{}^* du^* dv^*\right). \tag{26}$$

The solution of these coupled equations has been worked out by several authors (Adlam and Allen 1958; Davis, Lüst, and Schlüter 1958). We shall here present the solution due to Gardner and Morikawa (1965). If the local velocity of sound and the Alfvén velocities are called

$$c_s^2 \quad \text{and} \quad a_A^2\,,$$

the solution to be presented is valid when

$$\epsilon^2 = M^2 - 1 \tag{27}$$

is a small number. Here M^2 is the Mach number defined as

$$M^2 = \frac{U_0^2}{c_s^2 + a_A^2}\,. \tag{28}$$

Defining now the new variables

$$x = \epsilon \frac{eB_0}{cm_+ U_0} x^*\,, \quad v^* = U_0 v\,, \quad u^* = U_0(1 + U)\,, \quad B^* = B_0[1 + \epsilon^2 B]\,, \quad f_\pm{}^* = \frac{U_0^2}{n_0} f_\pm\,, \tag{29}$$

and expanding the distribution function f in powers of ϵ, i.e., writing

$$f = f_0 + \epsilon f_1 + \epsilon^2 f_2 + \cdots\,, \tag{30}$$

Gardner and Morikawa were able to show that the differential equation satisfied by $B(x)$ is given by

$$B_x - 3BB_x - RB_{xxx} = 0\,, \tag{31}$$

where the parameter R is given by

$$R = \alpha + \frac{(1 - 12\alpha)(\beta_+ - \alpha\beta_-)}{8(1 + \beta_+ + \beta_-)} + \frac{12}{16}\frac{(1 + \alpha)(\beta_+{}^2 + \alpha\beta_-{}^2)}{(1 + \beta_+ + \beta_-)^2} - \frac{9}{16}\frac{(\beta_+ - \alpha\beta_-)^2}{(1 + \beta_+ + \beta_-)^2}\,, \tag{32}$$

$$\alpha = m_-/m_+\,, \qquad \beta = \frac{nkT}{B^2/8\pi}\,.$$

The differential equation satisfied by $B(x)$ is just the special case of the KdV equation studied before, equation (13), with

$$\alpha = 3v\,, \qquad \beta = Rv\,. \tag{33}$$

The solution, equation (16), is then ($\beta = 1$)

$$B(x) = \mathrm{sech}^2\,(x/2R^{1/2})\,, \tag{34}$$

so that to the lowest order in ϵ^2 we obtain

$$B^*(x) = B_0[1 + \epsilon^2\,\mathrm{sech}^2\,(x/2R^{1/2})]\,. \tag{35}$$

Since the magnetic energy is proportional to $B^2/8\pi$, we can interpret the solution as indicating that there is a lump of magnetic energy well isolated in space. At the same time one can see from Gardner and Morikawa's analysis that the density of particles differs from the uniform Maxwellian distribution n_0 by a factor proportional to $B(x)$: *a space inhomogeneity has been generated.* Evidently the strength of the hydromagnetic soliton is B_0 since the transformation of variables has made $B(x)$ dimensionless. We shall then write equation (35) as

$$B^*(x) = B_0 + B_1(x)\,, \tag{36}$$

$$B_1(x) = B_{10}\,\mathrm{sech}^2\,(x/2R^{1/2})\,, \tag{37}$$

where

$$B_{10} = \epsilon^2 B_0\left(\frac{U_0{}^2}{c_s{}^2 + a_A{}^2} - 1\right)\,, \tag{38}$$

i.e., the amplitude of the soliton depends on the initial magnetic field B_0. As we have seen before equation (1), the value B can be as high as 10^{13} gauss at $t \approx 1$ s, enough to offset the smallness of ϵ^2 and therefore make the strength still appreciable. Fortunately, the previous theory does not depend upon the assumed value of B_0.

The underlying idea of this model is that the *perturbation $B_1(x)$ can now be used as an initial condition in the KdV equation,* in the same spirit as equation (17) was used as an initial condition in the Zabusky-Kruskal analysis discussed before.

Unfortunately, however, we do not yet possess a KdV equation derived from cosmological considerations. We can only give arguments that such an equation will emerge from the Einstein equations, when Lifshitz's analysis will be generalized to include nonlinear effects. Although the analysis is still missing, the result must be highly nonlinear since both the Einstein equations and the energy momentum tensor of matter are highly nonlinear. We can say that nonlinearity is the rule and linearity is the exception.

It could well be that the final result of an analysis à la Lifshitz, taking higher order terms into account, will lead to a nonlinear equation different from the KdV structure. This would however be of no great importance, since we have already shown other nonlinear equations that admit solitons.

The idea of having an original hydromagnetic wave traveling into a plasma and giving rise to a soliton solution is just one specific mechanism of how one can generate solitons.

The example given above is an extension to the case of warm plasma of an original work by Adam and Adler who studied the case of cold plasma. Figure 2*d* of their paper representing

$$\delta(x) + 1 = n(x)/n_0\,, \tag{39}$$

i.e., the ratio of the density to the unperturbed density, is particularly illustrative. Depending on the value of

$$U_0/U_A\,,$$

the increment $\delta(x)$ can be very significant, i.e., *large-scale inhomogeneities can indeed be generated.*

We shall conclude this section by pointing out that there are many good reasons to expect the generation of solitons in the early epochs of the Universe, where densities were arbitrarily large and nonlinear effects therefore particularly significant. So far, however, the published investigations of those early stages of the Universe have not viewed the problem under this light, and we can only hope that such an analysis will soon be undertaken.

IV. SECOND MODEL

In this second model we shall discuss a specific mechanism whereby radiation and matter could have kept separate from one another, rather than being thoroughly mixed.

The process of mixing radiation and matter before t_D is usually presented on intuitive grounds more than with any specific proof. It is conceivable that such a mixing occurred only at some later stages and that it only describes the gross features of the early Universe, leaving room for fine-scale phenomena of different type.

During the long period of time before t_D, we can only rely on nucleosynthesis as a rather firm point occurring at $T \approx 1$ MeV. Before that, we can only present a plausible scenario that has as landmarks the temperatures of 1 GeV, 100 MeV, . . ., when $N\bar{N}$, $\mu\bar{\mu}$, etc., annihilated irreversibly.

The irreversible annihilation of N and $\bar{N}$ occurred at $\sim 10^{-6}$ s. Before that, the annihilation processes of baryons heavier than nucleons produced radiation in the 1–10 GeV region. Since the spectrum of excited baryons seems to be exponentially growing with mass, we can in first approximation look at a very dense system of baryons in thermal equilibrium with a gas of photons, all with energies between 1 and 10 GeV. That is just about all we can say concerning radiation at those early epochs. The existence of a 3 K (seemingly) blackbody radiation spectrum today cannot be used to imply that the spectrum was also of blackbody type at any time earlier than t_D, $z \approx 10^3$.

It is only for logical simplicity that one extrapolates the blackbody nature of the spectrum back to times earlier than 10^6 years.

The phenomena we are about to discuss in this second model rely solely upon the existence of a dense gas of almost equally energetic photons. Such photons certainly existed before 10^{-6} s, when all the existing hadrons were in equilibrium with radiation. This is not meant to imply that we are denying the possible formation of a blackbody spectrum even at those earlier times. We are simply saying that our model relies on the existence of a block of photons with quasi-uniform energies.

Clearly the expansion of the Universe implies a rather strong fractional variation $\dot{\lambda}/\lambda$ of the wavelengths under consideration.[1] However, what is implied here is that *at a given time* there was an almost equally energetic gas of photons and not that the situation continued forever. The model to be presented only implies that at a certain given time such a situation occurred. The separation of matter and radiation occurred at that very time only; and, as shown explicitly in the text, it could not have occurred at much later times, the physical situation having changed too drastically for the mechanism to be still operative.

What about the second condition, $\nabla I(r) \neq 0$?

There are several ways of looking at it. The simplest is just to assume it and to investigate the consequences. Since it will turn out that they are extremely helpful in sorting out several deadlocked situations, one can feel that this is enough of *a posteriori* justification for its validity.

Second, we can see whether the condition $\nabla I(r) \neq 0$ has unphysical implications even before any cosmological consequences are considered. It will be shown, equation (47), that the length over which $I(r)$ varies, say l, must be large compared with $\lambda V_m^{1/2}$, where V_m is the maximum value of the repulsive potential barrier, in units of mc^2, built up by the photon gas against matter.

Since it will be shown that $V_m = t_m/t$, where $t_m = 10^{-q}$, with $q = 3$, 7, and 9 for electrons, muons, and nucleons, respectively, the condition is therefore

$$l > \lambda(t_m/t)^{1/2} .$$

Since $\lambda \sim T^{-1}$ and $T = 10^{10}t^{-1/2}$, the condition

$$l > 10^{-(10+q/2)} ,$$

with

$$l_{\min} = 10^{-(10+q/2)} ,$$

follows. Since, on the other hand, the maximum scale is ct, we have for l the upper and lower bounds

$$ct \geq l \geq 10^{-(10+q/2)}$$

or

$$t > 10^{-(20+q/2)} \text{ (seconds)}$$

—a condition easy to satisfy.

Finally if one accepts the suggestion of Ozernoi and collaborators that the photon gas was initially turbulent, it follows almost by definition that $\nabla I(r) \neq 0$.

In fact, one knows that in any turbulent medium the large eddies not only contain almost the whole bulk of energy but also have the characteristics of being "permanent." Permanence of the large eddies means that if they were created with an anisotropic distribution of energy, such a feature will persist as time goes on. Since we want to deal not with the equilibrium isotropic Kolmogorov eddies, but with the energy containing part of the spectrum, i.e., large eddies, the previous argument applies.

We shall henceforth consider a mixture of photons, part of which have an almost monochromatic spectrum and an intensity $I(r)$ that varies with position.

We shall show how such a hypothesis leads to consequences that, aside from not contradicting anything so far accepted, engenders a series of facts that help to explain several empirical facts regarding galaxies.

Finally, it will be shown how matter turbulence can be generated via a mechanism well known in the laboratory, thus relieving the so-far ad hoc hypothesis about its origin.

[1] We owe this consideration to the referee.

In § IV*a* we shall show how the classical equation of motion of an electron embedded in a photon field with $\nabla I \neq 0$ exhibits the existence of a repulsive potential $V(r)$ proportional to $I(r)$ itself. In § IV*b*, the repulsion of matter by a photon gas will be studied by stressing the analogy with the phenomenon of photon repulsion by a dense electron gas. In § IV*c*, the condition for the existence of such a phenomenon will be studied in the appropriate cosmological context.

In §§ IV*d* and IV*f*, the evolution of those portions of matter separated from radiation referred to in what follows as cosmological protogalaxies (CPG), will be studied on the basis of the most recent knowledge about the behavior of matter at high density. It will be shown that the conditions are appropriate for the interface between them and the surrounding radiation field to become unstable, thus giving rise to a turbulent layer.

a) The Body Force

Let us consider the equation of motion of a charged particle in the field of a plane electromagnetic wave of the form

$$\begin{aligned} \boldsymbol{E}(r) &= E_0\boldsymbol{n} \exp [i\omega(t - \boldsymbol{n}\cdot\boldsymbol{r}/c)] \equiv E_0\boldsymbol{n} \cos \phi , \\ \boldsymbol{B}(r) &= \boldsymbol{n} \times \boldsymbol{E}/c . \end{aligned} \tag{40}$$

The solution of the relativistic equation of motion

$$\frac{d}{dt}(m_0\gamma\boldsymbol{v}) = e\left(\boldsymbol{E} + \frac{1}{c}\boldsymbol{v} \times \boldsymbol{B}\right) \tag{41}$$

for the $n = 0, 0, 1$ case is known to be (Clemmow and Dougherty 1969)

$$x = -\gamma_1 \frac{eE}{m_0\omega^2} , \qquad \gamma_1 = \gamma(1 - \beta_3) , \qquad y = 0 , \qquad z = -\gamma_1{}^2 \frac{e}{m\omega^2}\frac{eE_0 \sin \phi}{4m_0\omega c} . \tag{42}$$

Let us now suppose that the intensity $E_0{}^2$ is a function of r, and let us write

$$E_0(r) = E_0 + \boldsymbol{r}\cdot\nabla E_0(r) . \tag{43}$$

If we use for r the previously determined values, it is easy to see that the force along the x and z directions are, up to the second order in E^2,

$$\begin{aligned} f_x &= -\gamma_1{}^2 \frac{e^2}{2m_0\omega^2}\frac{dE^2}{dx} , \\ f_z &= -\gamma_1{}^2 \frac{e^2}{2m_0\omega^2}[\beta_x(1 - \beta_3)^{-1}]\frac{dE^2}{dx} . \end{aligned} \tag{44}$$

Calling Γ the combination of relativistic factors, one can write in general

$$f = -\tfrac{1}{2}\Gamma m_0c^2\nabla s^2(r) \equiv -\nabla V(r) , \qquad V(r) = \tfrac{1}{2}\Gamma m_0c^2 s^2(r) , \qquad s^2(r) = \frac{e^2E^2}{m_0{}^2\omega c^2} . \tag{45}$$

Equation (45) states that the effect of the spatial variation of $E_0(r)$ is that of producing a repulsive potential that pushes matter away from regions of high intensity.

The quantity $s^2(r)$ can alternatively be written as

$$s^2 = \frac{2}{\pi}\frac{e^2\lambda^2 I}{m_0{}^2c^5} = \frac{1}{4\pi^2}\frac{e^2\lambda^2E^2}{m^2c^4} ; \qquad I = h\nu cN_\lambda = cE^2/8\pi ; \tag{46}$$

or equivalently

$$s^2 = \frac{2}{\pi}\left(\frac{e^2}{m_0c^2}\right)\left(\frac{\hbar}{m_0c}\right)\lambda N_\lambda ,$$

where N_λ is the number of photons per cm^3 with wavelength λ. Clearly, for equation (43) to be meaningful, the length l over which the intensity varies must be large compared with r,

$$l > \frac{eE}{m\omega^2} = \frac{\lambda}{2\pi} s . \tag{47}$$

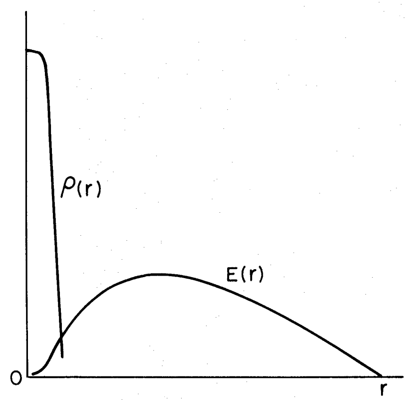

FIG. 1.—The spatial behavior of the electric field and particle density as solutions of Maxwell's equations. The concentration of particles is minimum where the radiation field is more intense.

When the previous argument is applied to a gas of particles of density ρ, the bodyforce exerted upon them is (Boot, Self, and Shersby-Harvie 1958)

$$F = -\tfrac{1}{2}\rho c^2 \nabla s^2(r) . \tag{48}$$

For static equilibrium to be attained, F must be balanced by a pressure gradient, i.e., $F = -\nabla p$. Using the equation of state of matter in the general form

$$p = c_s^2 \rho , \tag{49}$$

where c_s is the velocity of sound (in units of c), the hydrostatic equilibrium equation is easily integrated with the result

$$\rho(r) = \rho_* \exp\,[-s^2(r)/2c_s^2] . \tag{50}$$

The density of matter is lowest where the radiation density is the highest.

Using equation (50), one can derive the dielectric constant

$$1 - \epsilon = \frac{\rho_*}{\rho_c} \exp\,(-s^2(r)/2c_s^2) . \tag{51}$$

Maxwell's equations now read

$$\boldsymbol{\nabla} \times \boldsymbol{E} = i\boldsymbol{B}/\omega c , \qquad \boldsymbol{\nabla} \times \boldsymbol{B} = -i\omega\epsilon \boldsymbol{E}/c . \tag{52}$$

The qualitative behavior described by equation (50) can be quantified only after Maxwell's equations have been solved and the corresponding $E(r)$ versus r function determined. This in turn will yield $s^2(r)$ and finally the function $\rho(r)$. This set of highly nonlinear differential equations was first solved for a simplified geometry by Boot, Self, and Shersby-Harvie, and the results indicate that matter is pushed away from regions where $E^2(r)$ is maximum. The qualitative behavior is represented in Figure 1 where it is clearly seen that matter is confined to a little slab where the electric field is the smallest.

In conclusion we may put forward the following picture: in a first stage characterized by an anisotropic ensemble of large photon eddies, a strong force existed that pushed matter away from regions of high fields. The magnetic field behaved like a frozen-in field since it followed rather closely the behavior of matter, also being pushed away from regions of high $E^2(r)$.

We shall show in the next section that until enough time has elapsed for $s^2(r)$ to become small, the bulk of matter could not penetrate the surrounding dense radiation, much in the same way as a photon cannot penetrate into an electron gas if the latter is dense enough.

b) High-Intensity Fields

Consider an electron interacting with an electromagnetic wave of frequency ω, described by

$$E = E_0 \cos \omega t . \tag{53}$$

If we neglect the term $(e/c)\boldsymbol{v} \times \boldsymbol{H}$ in the equation of motion for the electron, we easily obtain the Thomson scattering cross section

$$\sigma = \frac{8\pi}{3}\left(\frac{e^2}{mc^2}\right)^2 . \tag{54}$$

Equation (54) is valid if $(e/c)\boldsymbol{v} \times \boldsymbol{H} \ll e\boldsymbol{E}$, or (remembering that the maximum attainable velocity is $v \sim eE/\omega m$) when the parameter (Kibble 1969, 1966; Reiss and Eberly 1966; Nikishov and Ritus 1964; Sarachik 1969)

$$s^2 = \frac{1}{\pi}\frac{e^2}{mc^2}\frac{1}{mc^3}\lambda^2 I = 3.10^{-11}\lambda^2 I\,,$$

$$s^2 = \frac{1}{\pi}\frac{e^2}{mc^2}\frac{h}{mc}\lambda N_\lambda\,, \tag{55}$$

is less than unity. This is the same parameter introduced in equation (45). We may note that for an ordinary light bulb, $s^2 \approx 10^{-18}$.

An alternative way of obtaining equation (55) and, in particular, the phenomenon of repulsion of an electron by a photon gas, is based on an analogy with the mirror process, namely the interaction of an electromagnetic wave with a dense electron gas. In that case, the relation between ω and k is known to be

$$\omega^2 = c^2k^2 + \omega_p{}^2\,, \tag{56}$$

where the plasma frequency is given by

$$\omega_p{}^2 = 4\pi e^2 N_e/m\,. \tag{57}$$

The refractive index $n = ck/\omega$ becomes imaginary when $\omega < \omega_p$, i.e., any such electromagnetic wave will be totally reflected by the electron gas. One usually says that a photon traveling in an electron plasma acquires a mass

$$m_\gamma{}^2 = \hbar^2\omega_p{}^2 c^{-4} = 4\pi(e\hbar/c)^2 N_e/mc^2\,. \tag{58}$$

Analogously, an electron traveling within an intense photon gas acquires an extra mass given by

$$m^{*2} = m^2 + 4\pi(e\hbar/c)^2 N_\gamma/\hbar\omega\,, \tag{59}$$

where we have substituted N_e with N_λ and mc^2 with $h\omega$ in going from (58) to (59). Using the definition of the parameter s^2, the effective electron mass can be written as

$$m^{*2} = m^2(1 + s^2)\,. \tag{60}$$

Using this relation, the refractive index turns out to be

$$n^2 = 1 - s^2/\beta^2\,, \qquad \beta = v/c\,. \tag{61}$$

The condition for an electron to be able to penetrate a photon medium is therefore

$$s^2 < \beta^2\,. \tag{62}$$

We shall show the cosmological period during which such condition is not satisfied. However, before doing so, we shall derive s^2 in an alternative way.

Consider the Lagrangian describing the interaction of an electron with a photon field:

$$\mathscr{L} = -\hbar c\bar{\psi}\gamma_\mu\partial_\mu\psi - mc^2\bar{\psi}\psi - j_\mu A_\mu/c\,, \tag{63}$$

where

$$j_\mu = iec\bar{\psi}\gamma_\mu\psi\,.$$

Since ψ satisfies the Dirac equation, the current can be split in three parts (Gordon decomposition), two of them being independent of A_μ and the third one being

$$j_\mu{}^{(3)} = -\frac{e^2}{mc^2}\bar{\psi}\psi A_\mu\,.$$

Substitution of this term into (63) gives

$$\mathscr{L} = -\left(mc^2 + \frac{e^2}{mc^2}A_\mu{}^2\right)\bar{\psi}\psi - \frac{1}{c}(j_\mu{}^{(1)} + j_\mu{}^{(2)})A_\mu - \hbar c\bar{\psi}\gamma_\mu\partial_\mu\psi\,. \tag{64}$$

Since we are considering a gas with an exceedingly high number of photons, we can use the classical expression for A_μ, i.e.,

$$\langle A_\mu\rangle \sim (hc^2N_k/\omega)^{1/2}\,. \tag{65}$$

The approximation $A_\mu \sim \langle A_\mu \rangle$ can be interpreted as saying that the electron has acquired an extra mass given by the second term in the first set of parentheses,

$$s^2 = \frac{\pi}{6}\left(\frac{e^2}{mc^2}\right)\left(\frac{h}{mc}\right)\lambda N_\lambda . \tag{66}$$

It is worth noticing that if, instead of a dense photon gas and one electron, we had an electromagnetic wave and a dense electron gas, the second term of (64) (after averaging over the electron wave function by writing $\bar{\psi}\psi = N_e$) could be interpreted as the free energy of a Proca field:

$$-m_\gamma^2 c^2 \hbar^{-2} A_\mu^2 , \tag{67}$$

thus providing equation (58) for m_γ^2.

c) *Cosmological Framework*

In order to study the implications of (62) for cosmology, we must relate the temperature T to the expansion time t. By solving Einstein's equations for a photon gas with $p = \frac{1}{3}\rho c^2$, one obtains the following relation:

$$\epsilon = \rho_\gamma c^2 = aT^4 = \frac{3c^2}{32\pi G} t^{-2} . \tag{68}$$

Eliminating T in favor of t and substituting back in (66), one obtains

$$s^2 = t_m/t , \tag{69}$$

where the "mixing time" t_m is given by

$$t_m = \frac{1}{6\pi^2}\left(\frac{45}{256}\right)^{1/2}\left(\frac{e^2}{mc^2}\right)\left(\frac{h}{mc^2}\right)\frac{1}{l_p} = 7 \times 10^{-3}\left(\frac{e^4 h}{m^4 c^5 G}\right)^{1/2} ,$$

$$l_p = (hGc^{-3})^{1/2} . \tag{70}$$

The mixing time t_m depends only on fundamental constants, and it can therefore be rewritten in different ways exhibiting combinations of the several cosmological numbers. Numerically we have

$$\begin{aligned} &\text{Nucleons: } t_m = 8\,10^{-10}\ \text{s} ; \\ &\text{Muons: } t_m = 6\,10^{-8}\ \text{s} ; \\ &\text{Electrons: } t_m = 3\,10^{-3}\ \text{s} . \end{aligned} \tag{71}$$

Since $\beta \lessgtr 1$, a sufficient condition for $n^2 < 0$ is $s^2 > 1$, or equivalently

$$t < t_m . \tag{72}$$

The repulsive potential barrier $V(r)$, equation (45),

$$V(r) = \tfrac{1}{2}mc^2 s^2(r) ,$$

has a maximum, say V_m, at

$$V_m = mc^2 t_m/2t . \tag{73}$$

If, for some reason, the particle energy E was consistently greater than V_m, then nothing would happen since particles could just skim over the rugged background of the photon field.

However, V_m and E have a different time dependence:

$$V_m \sim t^{-1} , \qquad E \sim kT \sim t^{-1/2} , \tag{74}$$

and so there certainly is a time t^* before which $V_m > E$ and the particles are trapped between maxima of the photon field.

This would not actually mean much if matter had the possibility of diffusing sidewise into the photon medium. However, for $t < t_m$, this is not possible since $n^2 < 0$, equation (72).

In Figures 2 and 3 we have plotted the sequence of events. The parameters q and p are (3, 0), (7, 2), and (9, 3) for electrons, muons, and nucleons, respectively.

Finally it is convenient to rewrite equation (50) in such a way as to exhibit the time dependence of the density contrast between the matter separated from radiation and the reference density ρ^*. We have

$$\delta(t) = \rho/\rho_* - 1 = 1 - \exp(-s^2/2c_s^2) = 1 - \exp(-t_m/2tc_s^2) . \tag{75}$$

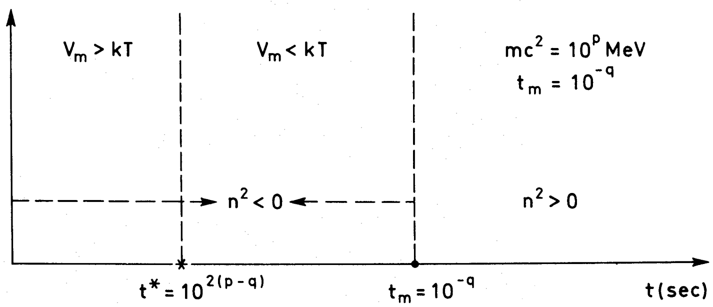

FIG. 2.—The repulsive potential V_m versus time. Up to $t = t^*$, $V_m > kT$ (the average energy per particle). The index q is defined in eq. (71). Only for $t > t_m$ the refractive index $n^2 > 0$.

Equation (75) implies that at very early times ($t \ll t_m$) we can obtain much higher density contrasts than at later stages, when $t \approx t_m$. Equation (75) *does not* represent the evolution in time of the density contrast; rather, it represents the density contrast that one can achieve at any given time t. To make this point clear, we should refer to the time evolution of a density contrast in the linearized theory of Lifshitz. In that case one does know that $\delta(t)$ grows with time like $\delta(t_i)f(t_i), f(t) \sim t^n$, t_i = initial time. The linearized theory of Lifshitz cannot predict the value of $\delta(t_i)$, which has to be computed by different means. In the present case, just the opposite is true. Equation (75) gives the function $\delta(t_i)$, i.e., at any chosen initial time we know $\delta(t_i)$. But since for $t < t_m$, $\delta(t) \sim 1$, the analysis of Lifshitz—or for that matter any linearized theory—is inapplicable, and the function $f(t)$ is therefore unknown. Finally we must also remark that the full problem is even more complicated than that since $s^2(r)$ also depends on time on the grounds that the Maxwell equations ought to be appropriately written to account for cosmological expansion.

d) *Stability of the Interface*

In order to render the mechanism that separates matter from radiation fully operative, we ought to show that no disruptive mechanism exists whereby the interface becomes so unstable as to generate a turbulent layer that will disrupt the whole system in a relatively short time. Since we do not know the full evolution of these portions of matter with respect to the surrounding medium, we shall study one case of instability which is general enough to be a good candidate. As we said earlier, the density of these portions of matter is higher than those of the surrounding medium.

Now, it is a known experimental fact that if the mixture of two substances is such that the density and pressure gradients are in opposite directions, the interface between them becomes unstable with an oscillation frequency given by

$$\omega^2 = (-\nabla p/p + \nabla\rho/\rho)\nabla p/p\,.$$

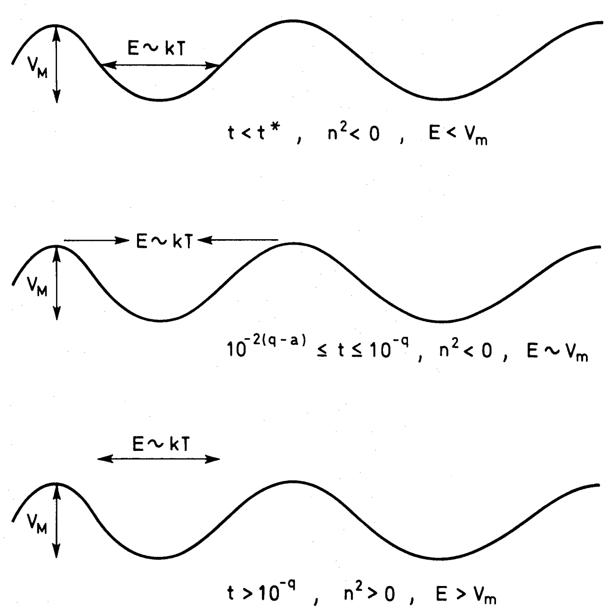

FIG. 3.—The same physical situation as in Fig. 1

A region of turbulent intermixing forms which is characterized by a length l_t and a velocity v_t related by (Belenskii and Fradkin 1967)

$$v_t = l_t|\omega| . \tag{76}$$

Equation (76) is the only quantity with dimensions of a velocity that one can build up using the previous expression or ω.

Calling L the region of intermixing ($\sim l_t$), the problem is reduced to that of finding the time-evolution of $L = L(t)$ and checking whether the region of turbulence grows in time fast enough to erode the whole CPG or if by virtue of the expansion of the Universe, the erosion process is actually slowed down and only part of the CPG is actually eroded. This rests on the fact that the driving force for the turbulent intermixing (the pressure gradient) is strongly affected by the stretching action of the expansion.

Calling c the concentration of the light substance, we have for the concentration equation

$$\frac{d}{dt}(\rho c) + \operatorname{div} \mathbf{j} = 0 , \qquad \mathbf{j} = -\rho \mathscr{D} \nabla c , \tag{77}$$

where $\mathscr{D}$ is the diffusion coefficient. Since we have shown, and it is experimentally known, that the intermixing gives rise to a turbulent motion, it would be incorrect in the definition of $\mathscr{D}$,

$$\mathscr{D} = v_t l_t , \tag{78}$$

to replace l_t by λ_{th}, the mean free path ($\sim 1/N\sigma_{\text{T}}$), and v_t by v_{th}. One must determine $v_t = v_t(l_t)$ from the theory of turbulence. This can be achieved by equating the expression for the amount of energy dissipated (per unit time) during the turbulent motion (an expression obtained by using the full set of hydrodynamic equations) to the phenomenological expression for the same quantity,

$$\epsilon_t = \int v_t^3 l_t^{-1} d^3 r ,$$

thus obtaining the desired relation,

$$v_t^2 = l_t^2\left[-\frac{1}{\rho^2}\nabla p \nabla c\left(\frac{\partial \rho}{\partial c}\right)_p\right] = l_t^2\left(-\frac{1}{\rho^2}\nabla p \nabla \rho\right) , \tag{79}$$

where the second expression is valid only for an incompressible substance:

$$\left(\frac{\partial \rho}{\partial c}\right)_p \nabla c = \nabla p . \tag{80}$$

In order of magnitude, equations (77) and (79) then give

$$\frac{\rho c}{t} \approx \frac{\rho l_t^2}{L}\left(\frac{1}{\rho} g \frac{1}{L} \rho\right)^{1/2} \frac{c}{L} , \quad L(t) = \alpha^4 g t^2 , \tag{81}$$

where the width of the turbulent region, L, has been taken equal to αl_t and where the driving force for the turbulent instability has been called g:

$$g \equiv -\rho^{-1}\nabla p . \tag{82}$$

The result is that the turbulent region increases quadratically with time. Relation (81) is valid for a nonexpanding Universe. To include the effect of expansion, one has to start the whole exercise from the beginning by evaluating

$$T_{\mu\nu;\nu} = 0 \tag{83}$$

where the semicolon stands for the covariant derivative. Using the definition of the energy momentum tensor,

$$T_{\mu\nu} = p g_{\mu\nu} + (p + \rho) U_\mu U_\nu , \tag{84}$$

and the Robertson-Walker metric, the Navier-Stokes equations and mass conservation are derived to be

$$\frac{\partial \boldsymbol{v}}{\partial t} + (\boldsymbol{v}\cdot\nabla)\boldsymbol{v} = -\frac{1}{R^2}\frac{\nabla p}{\rho} - f(t)\boldsymbol{v} , \quad \frac{\partial \rho}{\partial t} + \nabla\cdot(\rho \boldsymbol{v}) = H\rho , \qquad f = -2\frac{\dot{R}}{R} , \quad H = -3\frac{\dot{R}}{R} , \tag{85}$$

where we have taken the nonrelativistic limit after computing (84). The expansion factor is denoted by $R(t)$.

Equation (85) plus the concentration equation,

$$\rho\left[\frac{\partial c}{\partial t} + (\boldsymbol{v}\cdot\boldsymbol{\nabla})c\right] = -\boldsymbol{\nabla j} + H\,, \tag{86}$$

with $R = 1, f = H = 0$, were used to derive (79). The corresponding derivation with $R \neq 1, f \neq 0, H \neq 0$ is a long exercise and we have not yet succeeded in writing the final expression in a transparent form. For the moment, we shall therefore use a simplified argument. We note that the first term of the right-hand side of the Navier-Stokes equation, i.e., the driving force (82), has changed into

$$-\nabla p/\rho R^2 \tag{87}$$

in the case of an expanding Universe. The origin of the factor R^2 is clear. The pressure is a force per unit area and, as the time increases, the area increases as $R^2(t)$ and so the pressure decreases by the same amount. We shall therefore tentatively assume that (81) is still valid with $g(t)$ given by (84).

In the radiation-dominated universe, $R(t) \sim t^{1/2}$, and so the turbulent region will increase in time only as

$$L_{\exp}(t) \sim t\,.$$

In summary, we can say that the rather fast time increase of $L(t) \sim t^2$ has been slowed down and one can therefore hope that only part of the original CPG will be fluidized by the turbulent action during the cosmic expansion. Evidently, any quantitative statement has to await the full treatment of the turbulent intermixing in an expanding Universe.

V. RELATION OF THE PREVIOUS MODELS TO THE PROBLEM OF GALAXY FORMATION

That gravitational instability alone cannot account for galaxy formation is a well established result (Peebles 1973; Adams and Canuto 1975).

As we mentioned earlier, turbulence was proposed as a possible mechanism, and it looks like a very promising avenue indeed in spite of some recent difficulties. Even if they are overcome, it remains an ad hoc hypothesis.

Ozernoi and collaborators suggested that at earlier epochs the photon gas was turbulent. Matter tightly bound to radiation by Compton scattering (a hypothesis abandoned in this paper) was driven into a chaotic state; and when it decoupled from radiation at $R/R_0 \sim 10^{-3}$, it continued in that state of high vorticity.

The ad hoc hypothesis is removed in our second model since the existence of a separation mechanism between matter and radiation can give rise to a turbulent layer. Alternatively, we can say that instead of invoking turbulence as an *a priori* mechanism, it appears here as a phenomenon that cannot be dispensed with.

Even conceding that with the present model we could have a handle on the formation of turbulence, certainly alleviating a severe burden so far, still there is a long way to a satisfactory explanation of galaxy formation (Oort 1970*a*, *b*, *c*).

There seems to be no obvious candidate to supply the stirring action necessary to keep turbulence against decaying. Layzer (1964) has pointed out that the lifetime of turbulence against natural decay is shorter than the gravitational collapse time of an isolated cloud and this again is a serious difficulty.

Even admitting that this problem will be overcome some day, it remains unproven that the formulae valid for laboratory uncompressible turbulence could be applied to an expanding medium.

Finally, and this is perhaps the most severe obstacle, after t_D the velocity of sound has dropped to such a low value as to make the medium highly compressible, a physical situation for which we do not have a theory of turbulence. The application of Kolmogorov spectrum with variable index is a futile exercise. In fact, we know that Burgers equations for one-dimensional turbulence admit an exact solution that can be represented as a superposition of shock waves, and there is no trace of anything like an inertial spectrum à la Kolmogorov (Tatsumi and Kida 1972).

The drop in the velocity of sound is actually a blessing since it produces the necessary background for the formation of shock waves which will later pile up the material that goes into galaxies. In that sense turbulence is not the primary factor. It is important since it makes life for the shock waves much easier by generating high density contrasts. Whether the whole series of difficulties just encountered will be overcome in the future is not known. For that reason several people have sought different avenues. For a full appraisal of the current point of view, see Part IV of the IAU symposium, No. 63.

In one type of model a completely chaotic anisotropic initial universe is postulated. It seems, however, that several mechanisms of isotropization are actually at work in such a fashion that at times as early as 10^{-33} s the Universe is again isotropic.

Another possibility is related to the presence of delayed cores (Neeman 1965; Zel'dovich and Novikov 1965; Novikov 1965), chunks of matter that had a lower rate of expansion than the surrounding medium. The models suggested in this paper must be considered along these lines. They suggest physical phenomena that could lead to clumpiness. We have indicated the possibility of having a way of creating regions in which matter is denser than in others, this occurring because of high nonlinear phenomena.

Evidently the masses so created at $t \approx 10^{-9}$ s are too small to be identified with galaxies. In fact, the largest mass one can form at any time t cannot be greater than $M_H = 10^5 t(\mathrm{s})\, M_\odot$, the mass within the horizon. What is important, however, is not the mass per se, but the possibility of having strong density variation between that mass and the surrounding medium, thus giving rise to a deep gravitational potential well into which matter can subsequently fall, thus accreting onto the original one.

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